STRENGTH OF COMPOSITE MATERIALS

G. P. Cherepanov

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ABSTRACT: Here we consider the class of composites in which the strength of the contact between the materials is less than the strength of the components. It is found that the strength of such a material is independent of the size of the initial defect within certain limits but is determined by the shape and size of the most hazardous [weakest] inclusion. A theoretical relationship is deduced for the strength in relation to the size of the largest inclusion, which agrees well with experiment [1]. This mechanism probably plays a part in the failure of steel and may be one reason for the scale effect in steel.

§1. The scale effect. Consider a composite consisting of two materials. One of these (the bonding agent) is continuous, while the other forms inclusions. Boundary conditions occur at the interface between the two; we shall not consider their physical nature. The following basic assumptions are made.

(1) The inclusions are stronger than the bonding agent.

(2) The bond between inclusions and agent is weaker than both materials.

(3) The characteristic size of the most hazardous defect in the bonding agent (which determines the strength of the latter) is small relative to the characteristic size of the largest inclusion.

The detailed significance of these definitions will become clear from what follows. Of course, they apply only to some fairly general class of composites.

Consider such a material under uniaxial tension. The strength is determined by the most hazardous defect (a crack or dislocation). If the inclusions are stronger than the bonding agent, the stress tends to concentrate at the side of the boundary of an inclusion facing the tension direction. For instance, if the inclusion is a circular cylinder (Fig. 1), the tensile stress is largest at $\theta = 0$ (concentration coefficient 3/2 for a rigid circular inclusion). Elsewhere, the perturbation from the inclusion tends to reduce or even to reverse the stress.

From the second assumption above, the interface between the materials is the most likely site of failure, and we expect a crack to develop at that point, starting at $\theta = 0$ in Fig. 1 (the hazardous point); we assume that there is always some initial crack characteristic of the strength of the contact layer at that point. Growth of the initial crack is unstable, but dynamic growth generally soon ceases, and the crack is subsequently propagated along the interface as the load increases, with the stress coefficient increasing for the bonding agent near the end of the crack. The crack grows stably only up to a certain critical load, which corresponds to a limiting value of that coefficient [2]. Further growth of the crack (in the bonding agent) will be unstable if the inclusion is convex. If the inclusions are fairly evenly distributed in the body, and if the inclusion has a size not less than the order of the size of the other inclusions, then this critical load will be the limiting load for the whole body, i. e., will represent the strength

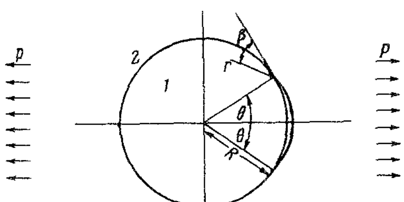

Fig. 1

of the composite. These arguments do not apply to materials whose inclusions take the form of long fibers if the tension is applied along the fibers. The strength of a composite that satisfies assumptions (1)–(3) is, in general, not dependent on the size of the initial crack but is determined by the shape and by the size of the most hazardous inclusion (a form of scale effect).

We therefore have to determine the strength σ of the composite in relation to inclusion shape and size d. The method of dimensions

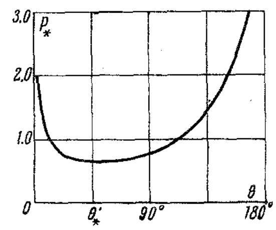

Fig. 2

readily provides the general form of the relation of σ to d. It is clear that σ is also dependent on the following quantities: the adhesion modulus K_2, the characteristic linear size l_0 of the limiting crack, the shear moduli μ_1 and μ_2, Poisson's ratios ν_1 and ν_2, and the ratio q of the contents of bonding agent and inclusions. (The subscript 1 refers to the inclusions, while the subscript 2 refers to the bonding agent.) Now l_0 is completely determined by D, the adhesion modulus for the contact layer between the materials, by d, and by K_2, μ_2, μ_1, ν_2, ν_1, and q. The product theorem gives

$$\frac{\sigma\sqrt{d}}{K_2} = F\left(\frac{\mu_1\sqrt{d}}{K_2}, \frac{\mu_2\sqrt{d}}{K_2}, \frac{D}{K_2}, \nu_1, \nu_2, q\right). \qquad (1.1)$$

It also follows from the stress problem that the stresses are dependent only on μ_1/μ_2, so the first two arguments in F appear as their ratio. The final result in dimensionless form is

$$\frac{\sigma\sqrt{d}}{K_2} = F\left(\frac{\mu_1}{\mu_2}, \frac{D}{K_2}, \nu_1, \nu_2, q\right). \qquad (1.2)$$

In particular, it follows from (1.2) that the hazard increases with the characteristic dimension of the inclusion, other things being equal. Staverman (see Fig. 79 in [1]) reported experiments on the stretching of rubber containing 42% NaCl crystals by volume:

$d =$	300—480	210—300	90—105	50—60	33—40 μm
$\sigma =$	4	5.2	9	12	13 kg/cm²
$\sigma\sqrt{d_{max}} =$	87.5	90	92	93	82.5

These results show that $\sigma(d_{max})^{1/2}$ differs from the mean of 89 by 7% in one instance and by 2–3% in the others, and therefore this composite belongs to the class envisaged here.

Formula (1.2) applies, provided that the size of the most hazardous inclusion is not comparable with the characteristic size of the most hazardous microcrack in the bonding agent; if the size of the inclusions is reduced below this limit, the strength of the composite will be independent of the inclusions (except, perhaps, for q close to unity).

§2. Example. Consider a single circular inclusion under the conditions of planar deformation of Fig. 1. An isotropic elastic body is represented in the complex z-plane by the exterior $(e^{i\theta}, e^{-i\theta})$ of a circle of radius R. The elastic constants of the inclusion ($|z| < R$) and bonding agent ($|z| > R$) are, for simplicity, taken as identical, while the strength of the contact layer ($|z| = R$) is taken as being less than the strength of the materials. The adhesion conditions apply at the boundary away from the crack. Let the surface of the cracks be free of external load and let the body at infinity be subject to uniaxial tension $\sigma_x = p$ (Fig. 1). The limiting equilibrium of a body with a crack in the form of an arc has been discussed [3]. We take a polar coordinate system $r\beta$ with its center at the end of the crack $z = e^{i\theta}$ ($\beta = 0$ corresponds to the extension of the crack along the tangent).

The stresses σ_r, σ_β, and $\tau_{r\beta}$ on the extension of the crack take [3] the following form for $r \ll R$:

$$\sigma_r = \sigma_\beta = \frac{k_1}{\sqrt{r}},$$

$$k_1 = \frac{p}{2\sqrt{2}}\sqrt{R\sin\theta}\left(\cos\frac{3}{2}\theta + \cos\frac{\theta}{2}\frac{1 - {}^1/_4\sin^2\theta}{1+\sin^2{}^1/_2\theta}\right),$$

$$\tau_{r\beta} = \frac{k_2}{\sqrt{r}},$$

$$k_2 = \frac{p}{2\sqrt{2}}\sqrt{R\sin\theta}\left(\sin\frac{3}{2}\theta + \sin\frac{\theta}{2}\frac{1 - {}^1/_4\sin^2\theta}{1+\sin^2{}^1/_2\theta}\right). \quad (2.1)$$

In the initial stage of growth, when the direction of propagation is known, the definitive characteristic of the material as regards failure is [4] $(\sigma_\beta^2 + \tau_{r\beta}^2)^{1/2}$, and, near the end of the crack, we have from (2.1) that

$$|\sigma_\beta + i\tau_{r\beta}| = \sqrt{\sigma_\beta^2 + \tau_{r\beta}^2} = \frac{p\sqrt{R}}{\pi f(\theta)\sqrt{r}},$$

$$f(\theta) = \frac{4\sqrt{2}}{\pi}\frac{3-\cos\theta}{\sqrt{\sin\theta\,(44 + 12\cos\theta + 12\cos^3\theta - 4\cos^4\theta + \sin^4\theta}}. \quad (2.2)$$

The condition for limiting equilibrium may [4] be put as

$$|\sigma_\beta + i\tau_{r\beta}| \to D/\pi\sqrt{r} \quad \text{for} \quad r \to 0, \quad (2.3)$$

in which D is a constant characterizing the resistance of the contact layer to failure, which is similar to the adhesion modulus for an isotropic and homogeneous body [2].

From (2.2) and (2.3) we have the following relation between the load p_* and the parameter θ defining the size of the crack:

$$p_* = f(\theta) \qquad (p_* = p\sqrt{R}/D). \quad (2.4)$$

Figure 2 shows $p_* = f(\theta)$. There is a region of instability for $\theta < \theta_*$ ($\theta_* = 45°$), while the crack grows stably for $\theta > \theta_*$. Let θ_2 be the point where σ_β at the end of the crack attains the limiting value K (adhesion modulus of the bonding agent). This θ_2 and the breakaway at $\beta = \beta_*$ of the crack occur when

$$\lim_{r\to 0}\sqrt{r}\,\tau_{r\beta}(\theta_2, \beta_*) = 0, \qquad \lim_{r\to 0}\sqrt{r}\,\sigma_\beta(\theta_2, \beta_*) = \frac{1}{\pi}K. \quad (2.5)$$

The size θ_0 of the initial crack governs the behavior of the elastic system.

Let p_{**} be the failure load.

1) $\theta_0 > \theta_1$ or $\theta_0 > \theta_2$, with $f(\theta_1) = f(\theta_2) = f(\theta_0)$. The initial crack does not grow until the load $f(\theta_0)$ is reached, after which failure is catastrophic. There is no scale effect.

2) $\theta_1 < \theta_0 < \theta_*$, with $p_{**} = f(\theta_2)$. When the load reaches $f(\theta_0)$, the crack grows dynamically to the value of θ corresponding to $f(\theta_0)$ on the stable branch of the curve. After this, the crack grows stably as p_* increases until θ_2 is reached.

3) $\theta_1 < \theta_0 < \theta_2$, with $p_{**} = f(\theta_2)$. The initial crack grows stably with the load from $f(\theta_0)$ to $f(\theta_2)$, whereupon failure occurs.

This shows that there is a scale effect when the initial crack satisfies $\theta_1 < \theta < \theta_2$, and the limiting load is not dependent on the initial size of the crack. The larger K/D the greater the range (θ_1, θ_2) and the more important the proposed failure mechanism, which implies a scale effect. If case (1) applies, the composite fails via onset of failure in the bonding agent. The length of the initial defect can be varied only conceptually, since this quantity is a characteristic of the material (contact layer or bonding agent) and is approximately constant. The above arguments indicate the limits to the size of the inclusions within which the strength is dependent on that size. A rough formula for these limits for $D \ll K$ is

$$\frac{2K^2}{\pi^2\sigma^2\theta_2} < R < \frac{2K}{\pi^2\sigma^2\theta_1}, \quad (2.6)$$

in which σ is the short-term resistance of the bonding agent.

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